The effect of wall shape on the degree of reinforcement of a shock wave moving into a converging channel

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The one-dimensional theory for the interaction between a normal shock wave and a discontinuous reduction in area is applied to the flow in a two-dimensional channel. It is shown that, for a given overall area ratio, a channel shape consisting of two equal discontinuous reductions in area produces a larger gain in shock strength than a channel having a single discontinuity. The gain in shock strength continues to increase with the number of steps, until the ideal limiting case of an infinite number of vanishingly small changes in area is reached.

A drum camera was used to measure, in a shock tube, the increase in speed of normal shock waves passing through two-dimensional converging channels of different wall shape. It was found that a single discontinuous change produced a gain in shock strength which was less than half that given by the one-dimensional analysis. However, the gain increased as the area change was made more gradual and for long smooth nozzles the value for the ideal case was nearly attained.

1. Introduction

It is not possible to analyse completely the two-dimensional flow associated with the motion of a strong shock wave through a channel of arbitrary wall shape as repeated Mach reflexion usually occurs (see, for example, Kahane, Warren, Griffith & Marino, 1954). However, if the assumption is made that the flow is one-dimensional, the interaction between a normal shock wave and a discontinuous area change can be analysed. Chisnell (1957) obtained a solution of this one-dimensional flow for a single infinitesimal change in area and then integrated this relationship to obtain the shock strength as a function of area. He applied this solution to the special cases of cylindrical and spherical shock waves and showed that the results given by this approach agree closely with those of Guderley (1942) and Butler (1954) which are exact for very strong waves.

The most general flow diagram for the interaction of a normal shock wave with a discontinuous reduction in area is shown in figure 1. This flow pattern is termed Type (a) and occurs when the flow in Region 2 is either supersonic or high subsonic, and the area ratio is too large for isentropic flow to occur through the area change without sonic conditions being reached. A strong reflected shock wave is produced and the flow behind it in Region 3 is subsonic, its velocity being fixed by the area ratio of the change, as the flow at the start of the reduced section 4 must be sonic. The flow then expands through a rarefaction fan which has its leading edge stationary at 4. A contact surface divides the flow consisting of gas which was originally in the wide section from that which was originally in the narrow section and a transmitted shock wave moves into the narrow section. When the flow behind the shock wave is supersonic, there is a limiting value of the area ratio below which the reflected shock wave is not sufficiently strong to move upstream. However, when this condition is reached, a modification to the flow pattern is possible, as the area ratio is small enough for

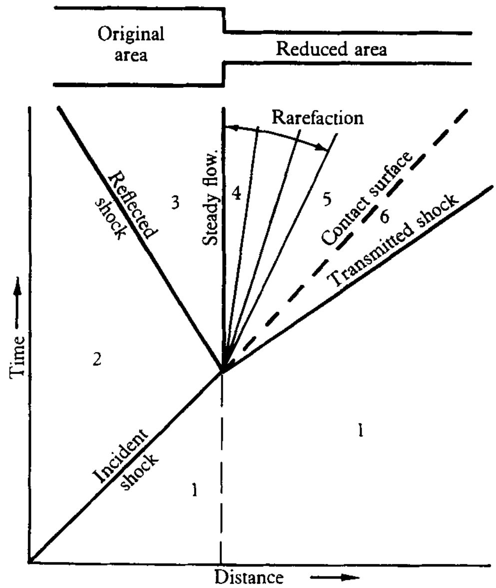


FIGURE 1. Wave diagram of general case of interaction between normal shock wave and discontinuous reduction in area.

steady isentropic compression of the supersonic flow in Region 2 to take place without critical conditions being reached in the area change. No reflected shock wave occurs and the rarefaction is carried downstream as the flow at the start of the reduced section is still supersonic. A contact surface and a transmitted shock wave are, of course, again formed and this flow pattern is termed Type (*b*) flow. There are combinations of shock strength and area ratio for which both types of flow are possible. A further modification which is called Type (*c*) flow occurs when the incident shock wave is weaker. The rarefaction fan is then not necessary and the flow in all regions, with the possible exception of the region between the contact surface and the transmitted shock wave, is subsonic.

A solution may be found for any particular problem by assuming one of the flow patterns, estimating the strength of one of the waves, writing down the relationships between the variables across the other waves and applying an iterative procedure until all the regions of the flow match and the boundary conditions are satisfied. Some results of this numerical analysis, which is similar to that used earlier on the same problem by Laporte (1954), are shown in figure 2, where the strength of the transmitted shock wave is plotted against the strength of the incident shock wave for interactions with area changes of a number of different ratios. Here, the strength of a shock wave has been defined as the pressure ratio across it. It can be seen that, when a shock wave of strength 7·0 is incident on an area change of ratio 10:1 a transmitted wave of strength 10·7 is produced. However, when the same wave meets a change of $\sqrt{(10)}:1$ a transmitted shock wave of strength 9·4 is produced, and when this wave meets another change of ratio $\sqrt{(10)}:1$ a final transmitted shock wave of strength 12·8 is produced. Therefore, when the reduction in area takes place in two equal steps the increase in shock strength is greater than if it takes place in one step of the same overall area ratio.

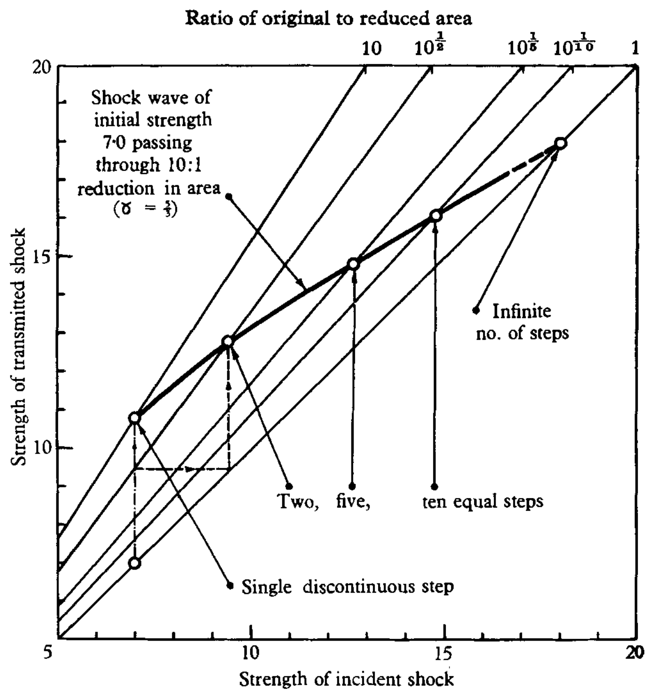


FIGURE 2. One-dimensional analysis of interaction between normal shock waves in a gas with a specific heat ratio of 5:3 and discontinuous reductions in area.

This general graphical procedure can be extended to any number of steps, and figure 2 shows the results for five and ten steps. The locus of these points may be extrapolated to the line of no area change and this gives a transmitted shock strength of 17·9. This corresponds to the ideal case of an infinite number of vanishingly small steps and is equivalent to Chisnell's analytical solution. Care must be taken when applying this graphical method to very strong waves, since the reflected shock wave in Type (a) flow causes non-isentropic losses which do not occur in Type (b) flow and the transition between the two is not continuous. In this example, a small region of Type (b) flow is possible for very small area ratios when the shock strength is above 9·27, but the reflected shock wave in the immediately adjacent region of Type (a) flow is sufficiently weak to be regarded as isentropic, so that the transition between the two regions is effectively continuous.

These considerations may be applied to the motion of a shock wave through converging channels of arbitrary wall shape, and it would be expected that the increase in shock strength for a given overall area change would be intermediate between that for a single discontinuous change and that for the 'ideal' channel. An experimental investigation was therefore undertaken to measure the speed and the strength of shock waves as they passed through two-dimensional converging channels of various wall shapes.

2. Experimental arrangements

The experiments were carried out in a shock tube using air as the high-pressure driving gas and argon as the low-pressure gas. The tube had a rectangular cross-section of 41 × 66 mm and the lengths of the high- and low-pressure sections were 0·75 and 3·0 m, respectively. Gas bottles were used to supply both sections, the low-pressure section being evacuated by a vacuum pump before the argon was admitted. The high and low pressures were set as near as possible to 2·4 and 0·032 atm., respectively, so that the nominal pressure ratio across the diaphragm was 75:1. Cellulose aceto-butyrate of 0·25 mm thickness forming the diaphragm was ruptured by remote control using a solenoid operated pricker. The theoretical shock strength for a diaphragm pressure ratio of 75:1 is 7·25 and the actual strengths ranged from 6·4 to 7·3, the scatter being primarily due to difficulties in accurately setting the pressure. The symmetrical nozzle blocks were carefully fitted in the working section which was 50 cm long and had windows which were 30 cm long and 1·2 cm wide. Nozzles 1–6 (table 1) were made of steel, while 7 was of polished wood; the area ratio of the contraction was 9·9:1 in all cases.

The image of a double mirror schlieren system was projected on to the film in a drum camera, so that a record of the flow was obtained in the distance–time plane. The light source for the schlieren was a water-cooled mercury arc lamp. As this provided a continuous source, an electromagnetically operated shutter was placed directly behind the slit so that the light could be allowed to pass for less than one revolution of the drum. A pulse generator in the camera produced 50 pulses each revolution of the drum; these were counted in the drum-camera control unit and the result was displayed on a frequency meter, which therefore gave a measure of the speed of rotation of the drum. This speed was set to the desired value by an adjustable transformer. A piece of thin aluminium foil was placed across the diaphragm and, when ruptured, provided a signal to the control unit. This signal initiated a time delay after which a pulse was passed to open the shutter. The shutter remained open until 16 pulses from the drum camera had been counted and then shut. The times were set so that the shutter was open and a record was produced by the camera while the shock wave was passing through the working section.

A number of the records obtained from the drum camera are reproduced in figure 3 (plate 1). The flow pattern in figure 3*a* is generally similar to that shown in figure 1 and the nature of the waves in this camera record may be deduced by comparing the two. The flow patterns in the other records are generally similar, although the discontinuities become more diffuse as the area change is made more gradual. The optical reduction of the schlieren system and the speed of rotation of the drum were known so that the speed, q, of the shock was easily

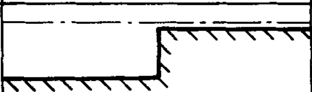
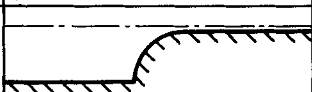
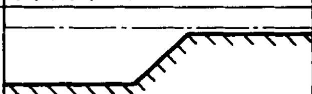
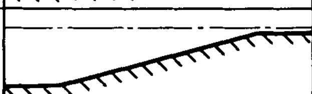
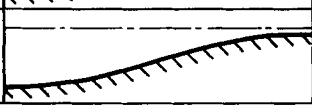
Nozzle blocks [Two-dimensional symmetrical area ratio 9·9 : 1]			Shock wave strength			
No.	Shape	Description	Incident	Transm.	% increase	Mean % increase
1		90°, sharp corner	7·3	9·1	25	25
2		90°, 0·1 H radius corner	7·1	9·1	29	29
3		90°, 0·2 H radius corner	7·0 7·2	9·3 9·6	33 33	33
4		Quadrant shaped	6·4	11·0	72	72
5		45°, straight	6·8 6·4	12·8 11·9	88 86	87
6		15°, straight	6·6 7·2 7·3	14·6 15·8 16·2	121 119 122	121
7		15°, rounded	6·9 7·3	16·1 16·5	133 126	130

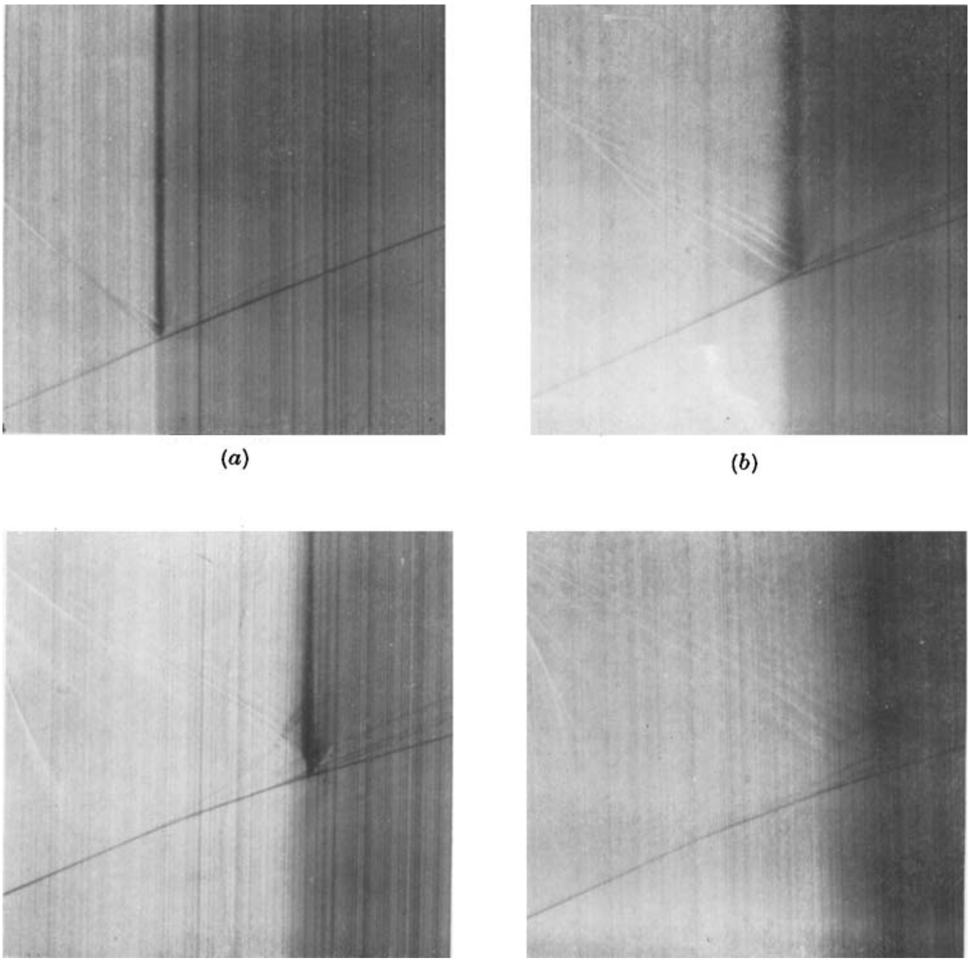
Table 1. Results of experiments on interaction between normal shock waves in argon and converging nozzles

obtained from the camera record. The speed of sound, a, in the undisturbed gas ahead of the shock, was also known and the strength, z, of the shock waves followed from the expression

$$z = \frac{2\gamma}{\gamma+1}\left(\frac{q}{a}\right)^2 - \frac{\gamma-1}{\gamma+1},$$

where γ is the ratio of the specific heats of the gas.

The initial change in the shock speed in the records for the 15° nozzles appears almost discontinuous as the windows showed only the portion of the shock at the centreline of the tube. No change in shock speed was therefore observed until the intersection point of the Mach reflexions from the top and bottom nozzles



(*a*) (*b*) (*c*) (*d*)

FIGURE 3 (plate 1). (*a*) 90°, 0·1 H radius corner nozzle blocks. (*b*) Quadrant-shaped nozzle blocks. (*c*) 15°, straight nozzle blocks. (*d*) 15°, rounded nozzle blocks.

met at the centre. The darkening of the background in the reduced area section of the records was caused by the nozzle blocks protruding over the edge of the windows.

3. Experimental results and discussion

The experimental results are summarized in table 1. It is estimated that the error in the strengths of the incident and transmitted shock waves was of the order of 1 %. In the cases where the tests were repeated, the results were consistent within the order of accuracy of the measurements. The variations in the incident shock strength are unimportant as the percentage increase in shock strength is not sensitive to them.

The 90°, sharp corner nozzle gave an increase in shock strength of only 25 % compared with the theoretical value (see figure 2) for a discontinuous area change of 53 %. This discrepancy can be accounted for as the theory assumes inviscid isentropic steady flow across the contraction and this could not possibly be approached in this nozzle. In addition, the shock would have to travel a distance equal to several tube widths past the change before this flow pattern could become established. It was noted on the record that the shock did not attain its final speed until it had moved some distance past the area change, its initial behaviour corresponding to an interaction with an area change of lower area ratio. Small radii of one-tenth and one-fifth of the height of the nozzle blocks increased the shock strength reinforcement to 29 and 33 %, respectively.

When the radius was increased to the full height of the nozzle blocks (i.e. quadrant shaped nozzle) the gain in shock strength rose to 72 % which is well above the theoretical value for a single discontinuous area change. The 45° straight nozzle which had the same area transition length as the quadrant nozzle produced a larger gain of 87 %. This indicates that the overall change in shock strength is sensitive to the initial strength of the reflected shock wave, which would be greater for the quadrant shaped blocks. For comparison, the theoretical values of the increase in shock speed for the two, five and ten equal area change cases are 83, 100 and 128 %, respectively.

When the transition region was greatly lengthened by the use of the 15° straight blocks the gain in shock strength rose to 121 %. In order to avoid the losses caused by the sharp corners on this nozzle it was then replaced by one with the same general shape, but with the corners rounded and the gain in shock strength rose to 130 %. This approaches the theoretical value of 156 % for the ideal case of an infinite number of vanishingly small area changes. Chisnell pointed out that this theory assumes that the effects of the reflected and re-reflected disturbances on the primary shock wave may be neglected and these experimental results seem to provide some further justification for this assumption.

4. Conclusions

The degree of reinforcement of a shock wave moving into a converging channel was shown to depend critically upon the wall shape and the extreme values of the percentage increase in shock strength for a contraction of a fixed area ratio differed by a factor of five. The most effective set of nozzle blocks produced a gain in shock strength, which was more than twice as great as the theoretical value for a single discontinuous area change, and approached that for the ideal case of an infinite number of vanishingly small area changes. This indicates that a converging section could well be incorporated in the design of those shock tubes in which the strongest possible shock waves are desired. Care would have to be taken in positioning the area change to ensure that the downstream travelling waves, produced by the interaction between the reflected waves from the area change and the contact surface from the diaphragm, do not overtake the primary shock wave in the working section.

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